**TDP (Honours) 4th Semester Exam., 2020**

PHYSICS

( Honours )

FOURTH PAPER (Group—A)

*Full Marks* : 48

*Time* : 2 hours

*The figures in the margin indicate full marks for the questions*

Answer **four** questions, taking **two** from each Unit

UNIT—I

**1.** *(a)* What are constraints? Give two examples of holonomic and non-holonomic constraints.

*(b)* State the D'Alembert's principle. What is its importance?

*(c)* The Lagrangian of a system is

$$L = \sum_i \frac{1}{2} q_i^2 + \sum_i A_i \dot{q}_i - o$$

where $q_i$'s are generalized coordinates, $A_i$ and $o$ are functions of $q_i$ and time but not of $\dot{q}_i$'s . Calculate the generalized momenta and the Hamiltonian of the system.





*(d)* Write down the Lagrangian of a particle moving under a central force. Obtain the equations of motion.

(1+2)+(2+1)+(1+2)+(1+2)=12

**2.** *(a)* Show that the Hamiltonian function is equal to the total energy for a conservative system.

*(b)* Find the principal moments of inertia of a right circular solid cylinder with its centre of mass as origin of the coordinate system. Also find the products of inertia.

*(c)* Show that moment of inertia for a uniform cube is same.

*(d)* What is the significance of Euler's angles?

4+4+2+2=12

**3.** *(a)* Derive the relative expression for kinetic energy and hence show that it reduces to classical value for $v / c << 1$.

*(b)* A reference frame $S'$ is in motion with respect to a fixed frame $S$ with a velocity $0{\cdot}9c$. A rigid bar of length $l$ is at rest relative to $S'$. If the bar makes an angle $60°$ with $x'$ axis, find the length of the bar and its orientation with respect to frame $S$.

*(c)* Show that no signal can have a speed greater than that of light to retain the order of events in two reference frames, one moving with respect to other with velocity $v$. 5+4+3=12

UNIT—II

*( Used symbols have their usual meanings )*

**4.** *(a)* A straight cylindrical hollow pipe carries a current of 100 A, has inner and outer radii 1·0 cm and 2·0 cm respectively. Using Ampere's circuital law, calculate the magnetic field at a distance of 0·5 cm, 1·5 cm and 4·0 cm from the axis of the pipe.

*(b)* Verify that the magnetic vector potential $\vec{A}$, due to uniform magnetic field $\vec{B}$ is given by

$$\vec{A} = -\frac{1}{2}(\vec{r} \times \vec{B})$$

*(c)* The electric field associated with an electromagnetic wave is

$$\vec{E} = \hat{x}E_0 \cos(kz - \omega t) + \hat{y}E_0 \sin(kz - \omega t)$$

where $E_0$ is a constant. Find the corresponding magnetic field $\vec{H}$ and the Pointing's vector $\vec{S}$.

*(d)* Explain briefly the Hertz's experiment with figure to generate the electromagnetic wave. 3+3+(2+1)+3=12





**5.** *(a)* Why is displacement current so called? How is Ampere's circuital law modified by its introduction?

*(b)* A light wave is always characterized by its electric field vector $\vec{E}$, although it has a magnetic vector $\vec{H}$. Explain why.

*(c)* Using Maxwell's equations, show that total electromagnetic energy is carried along the direction of propagation.

*(d)* A plane EM wave with electric field $\vec{E} = \hat{i}10\cos(kz - \omega t)\,\text{Vm}^{-1}$ is incident from air on a dielectric occupying the region $z \geq 0$. Determine the electric field of the reflected and transmitted waves. Given $\varepsilon = 4\varepsilon_0$ and $\mu = \mu_0$ for the dielectric medium. (1+2)+2+3+4=12

**6.** *(a)* What causes the attenuation of e.m. waves in a conducting medium?

*(b)* Find the state of polarization of the e.m. waves when the $x$ and $y$ components of the electric field are

*(i)* $E_x = E_0 \sin(\omega t + kz)$, $E_y = E_0 \cos(\omega t + kz)$

*(ii)* $E_x = E_0 \cos(\omega t + kz)$, $E_y = (E_0 / \sqrt{2}) \cos(\omega t + kz + \pi)$

(c) From Lorentz dispersion formula, derive the expressions for real and imaginary parts of refractive index in the context of anomalous dispersion in gaseous medium. Also show that the refractive index attains its maximum and minimum values at points where the absorption coefficient is half-maximum.

2+(2+2)+(3+3)=12

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(c) How do the superposition of two plane electromagnetic waves

$$\vec{E}_x = \hat{i} E_{ox} e^{j(kz-\omega t+\phi_x)} \text{ and}$$
$$\vec{E}_y = \hat{j} E_{oy} e^{j(kz-\omega t+\phi_y)}$$

result into an elliptically polarized electromagnetic wave? Mention the state of polarization of the superposed waves. 2+(3+3)+(3+1)=12

**6.** (a) Why is the quantity $\epsilon_0 \frac{\partial \vec{E}}{\partial t}$ referred to as displacement current?

(b) Considering the normal incidence of electromagnetic wave of frequency ω at the interface of two dielectric media, establish Fresnel's equations. Also show that the sum of reflectance and transmittance is equal to unity.

(c) Draw the variation of refractive index of a medium with wavelength indicating regions of normal and anomalous dispersion. 2+(4+4)+2=12

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# TDP (Honours) 4th Semester Exam., 2019

## PHYSICS

( Honours )

FOURTH PAPER (Group—A)

*Full Marks* : 48

*Time* : 2 hours

*The figures in the margin indicate full marks for the questions*

Answer **four** questions, taking **two** from each Unit

### UNIT—I

**1.** (a) Establish Hamilton canonical equation of motion. Also show that if a given co-ordinate is cyclic in Lagrangian, it will absent in Hamiltonian.

(b) Discuss the advantage of Hamiltonian approach over Lagrangian approach.

(c) Establish the principle of virtual work. Also explain how the forces of constraints have been avoided.

(d) A particle is moving on the surface of a sphere under gravity. Discuss about the nature of constraint. (3+1)+2+(2+2)+2=12

2. (a) A rigid body is rotating with an angular velocity $\vec{\omega}$ about an axis passing through origin and having direction cosines $(l, m, n)$. Find an expression for kinetic energy of rotation. How is the expression modified for principal axes system?

(b) Consider a system of two masses $m_1$ and $m_2$ suspended over a frictionless pulley and connected by a string of constant length. Find the Lagrangian and Lagrangian equation of the system.

(c) If $I_1, I_2, I_3$ be the principal moments of inertia of a rigid body, show that $I_1 \leq I_2 + I_3$. When does the equality sign hold good? (3+1)+(3+2)+(2+1)=12

3. (a) What are proper length and proper time interval?

(b) Prove that $E^2 = p^2c^2 + m_0^2c^4$ remains invariant under Lorentz transformation.

(c) Show that a photon cannot give rise to an electron-positron pair in free space in the absence of an external field.

(d) A rocket moves with a uniform velocity $V$ relative to the earth reaches a star in 1 years which is $L$ light years away (as measured by clocks at rest on the rocket). Show that $V = cL/(L+1)$. 2+4+3+3=12



( Continued )



UNIT—II

4. (a) Using Biot-Savart's law establish the differential form of Ampere's circuital law.

(b) Determine the magnetic vector potential at a distance $r$ from a very long thin straight wire carrying a current $I$. Hence find the corresponding magnetic field. Also check that $\vec{\nabla} \cdot \vec{A} = 0$.

(c) Two current loops of radii $R$ and $2R$ are coplanar and concentric. While the inner loop has a single turn, the outer loop has 8 closely wound turns. The same current $I$ flows through the two loops in the opposite sense. Find the magnetic field intensity close at the centre of the co-axial coils. 4+(3+1+1)+3=12

5. (a) Show that for a good conductor, skin depth, $\delta = \lambda_c / 2\pi$, where $\lambda_c$ is the wavelength of electromagnetic waves in a conductor.

(b) Starting from Maxwell's equation show that in a dielectric medium energy is equally shared between electric and magnetic field. Also show that in a good conductor, the field energy is almost entirely magnetic in nature.



( Turn Over )

(b) What is skin depth? The conductivity of a metal is $3.2 \times 10^{7}\ (\Omega\ m)^{-1}$. Find the attenuation of an electromagnetic wave of frequency 2 kHz by a metal plate of 0·1 mm thickness.

(c) Using Maxwell's equation, show that electric and magnetic field vectors together form a wave propagating with the velocity of light. (4+1)+(1+3)+3=12

6. (a) State Poynting's theorem.

(b) Calculate the amplitude of electric field produced by 100 W source at a distance of 10 m.

(c) A monochromatic light is incident on a gaseous medium containing $N$ molecules per unit volume. Find an expression for complex dielectric constant of the medium.

(d) Explain the modification made by Lorentz in Sellmeier's equation. 2+3+4+3=12

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**TDP (Honours) 4th Semester Exam., 2018**

PHYSICS

( Honours )

FOURTH PAPER (Group—A)

*Full Marks* : 48

*Time* : 2 hours

*The figures in the margin indicate full marks for the questions*

Answer **four** questions, taking **two** from each Unit

UNIT—I

1. (a) A pendulum with a bob of mass $m$ and length $l$, is suspended from a massless spring of constant $k$. The spring has only vertical motion. Find the Lagrangian and hence the Lagrange's equation.

(b) What is the advantage of using generalized co-ordinates?

(c) What is virtual displacement? Why is this concept introduced in mechanics?

(d) Write two basic characteristics of constraint forces. 5+2+(2+1)+2=12

2. *(a)* The three principal moments of inertia of a rigid body at a point are given by 20 kg-m$^2$, 30 kg-m$^2$ and 60 kg-m$^2$. Also the products of inertia are all zero. Calculate the moment of inertia of the body about an axis passing through that point and having direction cosines $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}\right)$.

*(b)* Give the geometrical interpretation of ellipsoid of inertia at a point.

*(c)* Use Hamilton's equation to obtain the motion of a particle of mass $m$ down a frictionless inclined plane of angle $\theta$.

*(d)* Show that if Hamiltonian of a system is time-independent, then it is a constant of motion. 4+3+3+2=12

3. *(a)* Simultaneity is not absolute but relative. Explain.

*(b)* Show that three-dimensional volume element is not invariant whereas four-dimensional volume element is invariant under Lorentz transformation.

*(c)* An electron and a positron practically at rest, come together and annihilate each other. Calculate the energy released.



( Continued )



*(d)* Give an example in support of mass-energy equivalence.

*(e)* The speed of an electron in a uniform electric field changes from 0·95 C to 0·98 C. Find the work done on electron due to change in velocity. 2+3+3+1+3=12

UNIT—II

4. *(a)* Show that Ampere's circuital law is not valid for non-steady current.

*(b)* Show that $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$, where symbols have their usual meanings.

*(c)* Find the vector potential due to a straight wire-carrying current $I$ and hence find the magnetic field.

*(d)* A wire-carrying current $I$ is bent in the form of a regular $n$-sided polygon. The distance of any vertex from the centre of the polygon is $r$. Find the magnetic field at the centre. Discuss the case when $n \to \infty$. 2+4+3+3=12

5. *(a)* Consider a plane electromagnetic wave travelling in a conducting medium. Find the electric and magnetic field vectors and their phase relations. What is meant by attenuation of the wave?



( Turn Over )

5. *(a)* What are exact and inexact differentials? Is work an exact differential? Give reason to your answer.

*(b)* Show that *J-T* expansion is an isenthalpic process.

*(c)* Show that *J-T* coefficient

$$\mu = \frac{1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right]$$

*(d)* State and explain Wien's law with necessary graph related to blackbody radiation. 3+3+3+3=12

6. *(a)* Show that the energy density of radiation inside a uniformly heated enclosure is given by $u = 4\pi k / c$, where $k$ is the specific intensity of radiation and $c$ is the velocity of light.

*(b)* What is meant by 'mean free path' of molecules in a gas? What is collision probability?

*(c)* Give the rigorous proof of principle of equipartition of energy. 4+(1½+1½)+5=12

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**TDP (Honours) 4th Semester Exam., 2017**

## PHYSICS

( Honours )

FOURTH PAPER (Group—A)

*Full Marks* : 48

*Time* : 2 hours

*The figures in the margin indicate full marks for the questions*

Answer **four** questions, taking **two** from each Unit

### UNIT—I

1. *(a)* If the three principal moments of inertia of a body at a point are $I_{xx}$, $I_{yy}$ and $I_{zz}$ respectively, then show that the equation of momental ellipsoid at that point is given by $I_{xx}x^2 + I_{yy}y^2 + I_{zz}z^2 = \text{constant}$.

*(b)* Calculate the moments and products of inertia of a right circular solid cylinder of mass $M$, radius $R$ and height $h$.

*(c)* Prove that the moment of inertia for a uniform cube is the same about any axis through the centre. 5+5+2=12

2. (a) If the equations of transformation do not depend explicitly on time and if the potential energy function $V$ is velocity independent, then show that the Hamiltonian represents the total energy of the system.

(b) The Lagrangian of a system having 2 degrees of freedom is given by $L = \dot{q}_2^{\,2} + 2\dot{q}_2\dot{q}_1 + \dot{q}_1^{\,2}$.

(i) Calculate the generalized momentum.

(ii) Set up the Lagrangian equation.

(iii) Obtain the Hamiltonian of the system.

(c) A particle moves on the surface of a sphere, whose radius is given by $R(t) = R_0 + \alpha t$, where $R_0$ and $\alpha$ are constants. Write down the equation of constraints. What type of constraint is this? 5+4+3=12

3. (a) What is the importance of the negative results of Michelson-Morley experiment?

(b) Prove that to a stationary observer, the moving clock appears to go slow.

(c) Show that the total relativistic energy, $E = mc^2$, where symbols have their usual meanings.

(d) The density of a body in a rest frame is $\rho_0$. Find the velocity of the moving frame in which the density is 10% higher. 2+3+4+3=12



( Continued )



UNIT—II

4. (a) Starting from Biot-Savart law, establish the relation $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$.

(b) Explain the physical significance of Maxwell's electromagnetic field equations.

(c) What is Poynting's vector?

(d) A proton at a distance 10 cm from a long straight wire carrying current 1 A is moving parallel to the direction of current flow with a speed 0·6c. Calculate the force on the proton. 3+4+2+3=12

5. (a) What is magnetic vector potential? Find the vector potential due to a straight wire carrying current $I$ and hence find the magnetic field.

(b) What is 'skin depth'? Show that for a conductor, it depends on the frequency of incident wave, whereas for a poor conductor, it is independent of the incident frequency.

(c) The intensity of sunlight heating the earth's surface is about 1300 $W/m^2$. Find the strength of the electric and magnetic fields of incoming sunlight. (2+3)+(1+3)+3=12



( Turn Over )

6. *(a)* Consider a plane electromagnetic wave incident normally on a dielectric medium. Find an expression for reflection and transmission coefficient in terms of refractive index of the medium.

*(b)* Starting from the expression for dielectric constant of the medium, obtain Cauchy's dispersion formula. Show graphically the variation of refractive index and absorption coefficient as a function of frequency of the incident electromagnetic wave.

*(c)* Find the reflectance and transmittance of a plane electromagnetic wave incident normally from air on a dielectric surface of refractive index 1·40. 5+(3+2)+2=12

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**TDP (Honours) 4th Semester Exam., 2016**

PHYSICS

( Honours )

FOURTH PAPER ( Group—A )

*Full Marks* : 48

*Time* : 2 hours

*The figures in the margin indicate full marks for the questions*

Answer **four** questions, taking **two** from each Unit

UNIT—I

1. *(a)* A rigid body rotates about an axis through the origin and has direction cosines ($l$, $m$, $n$). Show that the moment of inertia of the rigid body about the axis of rotation is given by

$$I_{lmn} = I_{xx}l^2 + I_{yy}m^2 + I_{zz}n^2 + 2I_{xy}lm + 2I_{yz}mn + 2I_{xz}ln$$

*(b)* What is ellipsoid of inertia? Show that in a conservative system, the Hamiltonian function is a constant and represents the sum of kinetic energy and potential energy.

(c) What is the significance of Euler's angles? 6+4+2=12

2. (a) Define virtual displacement. Show that the virtual work done by holonomic constraint forces is zero.

(b) A simple pendulum consists of mass $m_2$ with a mass $m_1$ at the point of support which can move on a horizontal line in the plane in which $m_2$ moves. Write the Lagrangian of the system and hence deduce Lagrange's equations.

(c) Prove that generalised momentum conjugate to a cyclic coordinate is constant in time. (2+3)+4+3=12

3. (a) Prove that four-dimensional volume element is invariant under Lorentz transformations.

(b) Calculate the speed of an electron which has kinetic energy 1·02 MeV. The rest mass of the electron is 0·51 MeV.

(c) Obtain Einstein's formula for addition of velocities and prove the principle of constancy of speed of light.

(d) Find the velocity if the mass of a particle is double to its rest mass. 3+2+5+2=12





UNIT—II

4. (a) Starting from Biot-Savart law, show that the divergence of magnetic field $\vec{B}$ is zero. What is its physical significance?

(b) Using magnetic vector potential $\vec{A} = e^{-x}\sin y\hat{i} + (1+\cos y)\hat{j}$, calculate the magnetic induction.

(c) Verify that the magnetic vector potential $\vec{A}$ due to uniform magnetic field $\vec{B}$ is given by $\vec{A} = -\frac{1}{2}(\vec{r} \times \vec{B})$. 12

5. (a) Starting from Maxwell's equations, derive the wave equation in a conducting medium. Hence show that a plane e.m. wave is attenuated as it propagates through the medium.

(b) State and establish Poynting theorem.

(c) An electromagnetic wave is passing through a non-conducting medium characterized by permittivity $10\varepsilon_0$ and permeability $5\mu_0$. Find the wave impedance. (3+3)+4+2=12