TDP (Honours) 1st Semester Exam., 2018

PHYSICS

(Honours)

FIRST PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **eight** questions, taking **two** from each Unit

UNIT—I

1. *(a)* State Green's theorem in the plane.

   (b) Verify Gauss' divergence theorem for the vector $\vec{R} = \hat{i}x + \hat{j}y + \hat{k}z$ taken over a unit cube in the first octant, i.e., $0 \leq x,\ y,\ z \leq 1$.

   (c) Show that the angular velocity of rotation of a rigid body is half the curl of the velocity vector of a particle within the body. 2+5+3=10

2. (a) Find the Fourier series to represent $f(x) = x - x^2$ from $x = \pi$ to $x = -\pi$. Then prove that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots\cdots\cdots = \frac{\pi^2}{12}$$

(b) If $\vec{\nabla} \times \vec{A} = \frac{\partial \vec{B}}{\partial t}$, then show that $\vec{\nabla} \cdot \vec{B}$ is independent of t.

(c) Show that the vector field

$$\vec{F} = 2xy\hat{i} + (x^2 + 2yz)\hat{j} + (y^2 + 1)\hat{k}$$

is conservative.

(d) What are curvilinear coordinates?

(4+1)+2+2+1=10

3. (a) Show that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$; where the symbols have their usual meanings.

(b) Show that any square matrix A can be expressed as a sum of one symmetric and one skew-symmetric matrix.

(c) Find the eigenvalues and the eigenvectors of the matrix $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. 4+2+4=10

UNIT—II

4. (a) Find the relation between angular momentum and moment of inertia.

(b) Show that the moment of inertia of a cylinder of radius a, mass M and height H about an axis parallel to the axis of the cylinder and at a distance b can be expressed as $I = \frac{1}{2} M(a^2 + 2b^2)$.

(c) Three identical particles of mass 0·01 kg each are placed at the vertices of an equilateral triangle of sides $\frac{3\sqrt{3}}{10}$ m. Calculate the moment of inertia of the system about an axis passing through the CG of the system and perpendicular to the plane of the triangle.

(d) Show that the acceleration of a body rolling down in an inclined plane is independent of the mass of the body.

3+2+2+3=10

5. (a) Prove that for the motion of a particle along a curve in a plane, the acceleration of the particle can be expressed as $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$; where the symbols have their usual meanings.

(Turn Over)

(b) A particle is acted upon by a central force describes an orbit given by the equation $r = a(1+\cos\theta)$, here a is a constant. Investigate the nature of the orbit and nature of the force.

(c) What will be the directions of the Coriolis force in the southern hemisphere and in the northern hemisphere? 3+(2+2)+(1½+1½)=10

6. (a) An object is thrown vertically upward with a velocity 70 m/s at a place with latitude 60°. Find how far from the original position it will land.

(b) Show that the total energy E of a particle of mass M acted upon by a central force can be expressed as

UNIT—III

7. (a) Calculate the depression at a point of a cantilever beam loaded at the free end, taking the weight of the beam into consideration.

(b) Assuming that the interior of the earth can be treated as homogeneous spherical mass in hydrostatic equilibrium, show that the pressure within the earth as a function of distance r is given by $P = \frac{2}{3}\pi\rho^2 G(R^2 - r^2)$ where R and ρ are the radius and mean density of the earth respectively.

(c) Explain why the use of hollow shafts is in preference to solid ones for transmitting large torques in rotating machinery.

4+3+3=10

8. (a) Show that a shear θ is equivalent to two equal linear strains of half the magnitude in mutually perpendicular directions.

(b) Derive a mathematical expression for the equation of continuity.

(Turn Over)

(c) State Bernoulli's theorem.

(d) Two spherical soap bubbles unite to form one bubble without any leakage of air. Assuming that the temperature remains unchanged during the process, show that $3P\Delta V + 4T\Delta S = 0$ where P is the atmospheric pressure and ΔV and ΔS are changes in volume and surface area respectively. 3+3+1+3=10

9. (a) Show that the internal bending moment of a beam is given by $\frac{YAK^2}{R}$; where the symbols have their usual meanings.

(b) Find the gravitational field intensity due to a solid sphere of radius R at a distance $2R$ from its centre. Given that the density inside the sphere varies with the distance r from its centre as $\rho = \frac{\rho_0 R}{r}$.

(c) A capillary tube of radius a and length l is fitted horizontally at the bottom of a cylindrical flask of cross-section A. Initially, there is water in the flask up to a height of h_1. What time would be required for the height to reduce to h_2, if η be the viscosity of water? 4+2+4=10

UNIT—IV

10. *(a)* Distinguish between amplitude resonance and velocity resonance.

(b) Show that for small damping, the average fractional loss of energy of a particle executing a damped simple harmonic motion in a cycle is four times the logarithmic decrement. Hence calculate the quality factor of a damped oscillator.

(c) Show that the resonant frequency is the geometric mean between the half power frequencies. 3+4+3=10

11. *(a)* Write the expression for the displacement $y(x, t)$ of stationary waves produced in a stretched string.

(b) Compare the intensities of the fundamental and the third harmonic if a stretched string fixed at the two ends is plucked at the midpoint.

(c) What happens if a string is plucked over a finite length instead of being plucked at a single point?

(d) What will be the effect on the emitted note from a plucked string if *(i)* the bridges supporting the string at the ends yield and *(ii)* the string is made thicker? 1+3+3+3=10

(Turn Over)

12. *(a)* Derive an expression for the growth of acoustic energy density with time in an enclosure. Plot the growth of sound intensity within the enclosure as a function of time.

(b) What is optimum reverberation time?

(c) A hall of volume 2500 m^3 has a total absorption equivalent to 300 m^2 of open window. There are 10 persons in the room each equivalent to 4 sabin absorption. What is reverberation time of the room?

(5+1)+2+2=10

★★★

TDP (Honours) 1st Semester Exam., 2017

PHYSICS

(Honours)

FIRST PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **eight** questions, taking **two** from each Unit

UNIT—I

1. *(a)* State Stokes' theorem on vector calculus.

   (b) Verify divergence theorem for the vector $\vec{A} = \hat{i}x^2 + \hat{j}y^2 + \hat{k}z^2$, taken over the cube $0 < x;\ y,\ z \leq 1$.

   (c) Prove that the spherical polar coordinate system is orthogonal.

   (d) Show that $\beta(m, n) = \beta(n, m)$. 2+2+3+3=10

(Turn Over)

2. *(a)* Verify Caley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

(b) Prove that $\vec{\nabla}\phi$ is a vector perpendicular to the surface $\phi(x, y, z) = c$, where c is a constant.

(c) Prove that the eigenvalues of a Hermitian matrix are all real.

(d) State Dirichlet's conditions for a Fourier series. 2+3+3+2=10

3. *(a)* Find a series of sines and cosines of multiples of x which will represent $x + x^2$ in the interval $-\pi < x < \pi$.

(b) Show that $\Gamma(n)\,\Gamma(n-1) = \dfrac{\pi}{\sin n\pi}$ and using this relation, find the value of $\Gamma(\frac{1}{2})$.

(c) State Green's theorem on vector calculus. 4+(3+1)+2=10

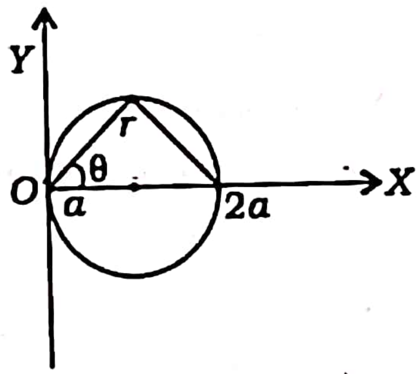
UNIT—II

4. (a) Find the moment of inertia of a uniform right circular cone of height h, radius r and mass M about its axis of symmetry.

(b) Determine the differential equation of motion of a particle moving under central force in plane polar coordinate system. 5+5=10

5. (a) Prove Newton's law of gravitation from Kepler's laws on planetary motion.

(b) A particle moves in a circular orbit which passes through O under the influence of central force at O as shown in the diagram. Find the law of force :



(c) Explain Coriolis force. 4+4+2=10

6. (a) Assuming the expression for the total energy in case of planetary motion, prove that $T^2 \propto a^3$, where symbols have their usual meanings.

(b) A river of width D flows northwards with a speed v_0 at a colatitude λ in the northern hemisphere. Prove that the difference H between water levels of the two banks due to Coriolis force is given by $H = \frac{2D\omega v_0 \cos\lambda}{g}$, where symbols have their usual meanings.

5+5=10

UNIT—III

7\. (a) State Gauss' theorem in gravitation.

(b) Using Gauss theorem, derive an expression for the gravitational field intensity at a point inside a solid sphere.

(c) Derive an expression for the torsional rigidity of specimen in the form of a cylinder of radius r and length l.

(d) Define fluidity. 2+3+4+1=10

8\. (a) Explain the term 'internal bending moment'.

(b) Derive an expression for the depression at the free end of a cantilever having circular cross-section, which is supported at one end and loaded at the other end.

(c) Discuss how Bernoulli's theorem is applied to measure the rate of discharge of water in a venturi meter. 2+4+4=10

9. (a) If the rate of change of surface energy of a liquid with temperature be proportional directly to the absolute temperature T, show that the surface tension is given by $S = aT^2 + bT + c$, where a, b and c are constants.

(b) State Jurin's law.

(c) Find the expression to determine the surface tension of a liquid by using sessile drop method. 3+2+5=10

UNIT—IV

10. (a) Two simple harmonic motions having same periods but different amplitudes and initial phases act simultaneously on a particle. Find the resultant motion. What happens when the phase difference between the motions are (i) 0, (ii) π and (iii) $\frac{\pi}{2}$?

(b) Write down the differential equation of motion of a simple harmonic oscillator subjected to a damping force and external simple harmonic force. Solve the differential equation. Explain critically damped motion. (3+2)+(1+3+1)=10

11. *(a)* Prove that the velocity of acoustic wave travelling along a solid rod is given by $\sqrt{\frac{Y}{\rho}}$, where Y is the Young's modulus and ρ is the density. Mention the assumptions made.

(b) A uniform string of length l is stretched between its fixed ends $x = 0$ and $x = l$. Obtain an expression for the transverse displacement $y(x, t)$ of the string when string is struck at any point on the string.

(c) From the equation of a wave, how can one judge that the wave is progressive or stationary? (3+1)+5+1=10

12. *(a)* Find an expression for distribution of pressure in longitudinal progressive waves.

(b) State Sabine's law. What are 'live room' and 'dead room'?

(c) Show that the energy density of a plane progressive wave is directly proportional to the square of the amplitude and also square of frequency of the wave.

3+(2+2)+3=10

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TDP (Honours) 1st Semester Exam., 2016

PHYSICS

(Honours)

FIRST PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **eight** questions, taking **two** from each Unit

UNIT—I

1. (a) If $\vec{r}(t)$ be a vector of fixed magnitude, then show that $\frac{d}{dt}[\vec{r}(t)]$ is perpendicular to $\vec{r}(t)$.

(b) Show that the vector

$$\vec{V} = (-4x - 3y + 4z)\,\hat{i} + (-3x + 3y + 5z)\,\hat{j} + (4x + 5y + 3z)\,\hat{k}$$

is irrotational. Also find the scalar function ϕ which can be expressed as $\vec{V} = \vec{\nabla}\phi$.

(c) Verify Stokes' theorem for

$$\vec{A} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$$

where S is the upper half-surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

2+(2+2)+4=10

2. (a) Explain with examples what is meant by a unitary matrix and a Hermitian matrix.

(b) Find the eigenvalues and eigenvectors of the following square matrix :

$$A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

(c) State Green's theorem in plane.

(2+2)+4+2=10

3. (a) Prove that cylindrical system is orthogonal.

(b) Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

(c) Expand $f(x) = x^2$ as a Fourier series in the interval $(-\pi, \pi)$ and hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots = \frac{\pi^2}{12}.$$

3+2+5=10

UNIT—II

4. (a) Find the moment of inertia of a uniform solid cylinder of radius r, length l and mass M about an axis through the centre of mass and perpendicular to the geometric axis.

(b) A solid cylinder of mass m and radius r is rolling down an inclined plane without slipping. Show that the speed of its centre of mass when the cylinder reaches the bottom of the plane is $2\sqrt{gh/3}$, where h is the height of the incline.

(c) What is radius of gyration? 5+3+2=10

5. (a) What are fictitious forces? How do they arise in a uniformly rotating frame?

(b) An object is allowed to fall under gravitational force from the top of an h metre high tower at the equator. Find the horizontal displacement of the object due to earth's rotation. (2+3)+5=10

6. (a) Show that the areal velocity of a particle moving under the action of a central force is constant.

(b) Show that gravitational force is conservative.

(Turn Over)

(c) Find the tangential and normal components of velocity and acceleration of a particle moving along a curve ABC in the XY-plane. 3+3+4=10

UNIT—III

7. (a) State and prove Gauss's theorem in gravitation.

(b) Define 'flexural rigidity'.

(c) Compare the loads required to produce equal depression for two beams, made of same material and having same length and weight, with the only difference that while one has a circular cross-section, the cross-section of the other is square. (1+3)+2+4=10

8. (a) Derive an expression for the depression at the middle of a bar of negligible weight supported horizontally at two ends and loaded at the middle.

(b) Stating clearly the basic assumptions, establish Bernoulli's theorem in fluid dynamics. 4+6=10

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9. (a) Show that the excess pressure acting on the curved surface of a curved membrane is given by

$$p = 2T\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

where r_1 and r_2 are the radii.

(b) State and prove Torricelli's theorem.

(c) What is 'torsional rigidity'? 4+4+2=10

UNIT—IV

10. (a) What is resonance? Show that at velocity resonance the maximum velocity is inversely proportional to damping factor. What is power factor at velocity resonance?

(b) A plane simple harmonic soundwave of amplitude 0·001 mm and frequency 650 Hz is propagated in a perfect gas of density 1·29 kg/m^3 and pressure 100 kPa. If $\gamma = 1 \cdot 41$, calculate acoustic pressure amplitude of the wave. (1+3+2)+4=10

11. (a) Explain group velocity and phase velocity.

(Turn Over)

(b) A wave group is formed by the superposition of two waves of equal amplitudes but of slightly different frequencies and wavelengths. Show that if c_g is the group velocity and C is the phase velocity, then $c_g = c - \lambda \frac{dc}{d\lambda}$.

(c) What are Lissajous' figures? 4+4+2=10

12. (a) Derive an expression for the growth of the acoustic energy density with time in an enclosure.

(b) Which aspects should be emphasized in the design of a good auditorium?

(c) What is optimum reverberation time?

5+3+2=10

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TDP (Honours) 1st Semester Exam., 2015

PHYSICS

(Honours)

FIRST PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **two** questions from each Unit

UNIT—I

1. (a) Show $\nabla^2 r^n = n(n+1)r^{n-2}$.

(b) State Gauss's divergence theorem and verify this theorem for $\vec{A} = \hat{i}4x - \hat{j}2y^2 + \hat{k}z^2$ taken over the cylinder bounded by the surfaces $x^2 + y^2 = 4$, $z = 0$, $z = 3$.

(c) Verify Green's theorem for

$$\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$$

where C is the boundary of the region R enclosed by the parabolas $y = x^2$ and $y^2 = x$.

2+(1+4)+3=10

2. *(a)* Determine the values of α, β, γ, when

$$\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$$

is orthogonal.

(b) Diagonalize the matrix

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

(c) What are curvilinear coordinates?

3+6+1=10

3. *(a)* Show that

$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

(b) Find the Fourier series for the periodic function

$$f(x) = \begin{cases} 0 \ , & -\pi < x < 0 \\ x \ , & 0 < x < \pi \end{cases}$$

5+5=10

UNIT—II

4. *(a)* On what factors does moment of inertia depend? What is the physical significance of moment of inertia?

(b) Find a relation between angular momentum and moment of inertia.

(c) A solid sphere of moment of inertia I about its diameter is converted in a disc of same moment of inertia I, about its axis. Find the ratio of their radii.

(d) Show that the acceleration of a body rolling down an inclined plane is independent of the mass of the body.

(1½+1½)+2+2+3=10

5. (a). Considering inverse square law, prove the Kepler's 3rd law.

(b) Prove that a central force is a conservative force and conservative force can be expressed as negative gradient of potential energy. 5+(2+3)=10

6. (a) Show that the differential equation of motion of a particle of mass m under the influence of a central isotropic force can be written as

$$\frac{d^2u}{d\theta^2}+u=\frac{m}{J^2u^2}F\left(\frac{1}{u}\right)$$

where $u=\frac{1}{r}$, (r, θ) are the plane polar coordinates of the particle and J the angular momentum.

(b) Distinguish between inertial and non-inertial frames of reference. Give one example of each. Is earth an inertial frame? Give reasons. 5+(3+2)=10

UNIT—III

7. (a) Applying Gauss's theorem, find out the expression of gravitational intensity at a point inside a solid cylinder and hence find potential at that point.

(b) Derive Poiseuille's formula for the rate of steady flow of liquid through a capillary tube of circular cross-section.

5+5=10

8. (a) Find the depression of a cantilever beam of uniform cross-section and weight w, when loaded at the free end by a weight w_0. What is the strain energy in this case?

(b) What is meant by torsional oscillation? Discuss how the modulus of rigidity of the material of a long wire can be determined using torsional oscillations.

(c) Bulk modulus of water is $2 \cdot 3 \times 10^5$ dy/cm^2. How much pressure is needed to compress a sample of water by 0·1%?

4+4+2=10

9. *(a)* Explain what you understand by surface tension of a liquid and state its unit.

(b) Deduce expressions for the excess of pressure inside a soap bubble.

(c) Find the energy expanded in spraying a drop of water of 1 mm radius into a million droplets all of the same size. Given that the surface tension of water is $7 \cdot 2 \times 10^{-2}$ N/m. 2+4+4=10

UNIT—IV

10. *(a)* What do you mean by Lissajous figures?

(b) What do you mean by dispersion in wave propagation?

(c) Write down the characteristics of a good Auditorium.

(d) Define group velocity with mathematical derivation.

(e) What do you mean by 'live room' and 'dead room'? 2×5=10

11. *(a)* Derive an expression for the intensity of a plane progressive, harmonic sound wave.

(b) State Young-Helmholtz law.

(c) If a string be pulled h cm, vertically at the centre, show that the displacement is given by

$$y = \frac{8h}{\pi^2}\left[\sin\frac{\pi x}{l}\cos\frac{\pi vt}{l} - \frac{1}{9}\sin\frac{3\pi x}{l}\cos\frac{3\pi xt}{l} + \cdots\right]$$

where the symbols have usual meaning.

3+2+5=10

12. (a) Set differential equation for damped simple harmonic motion. Solve the equation and state the condition for which the system ceases to oscillate.

(b) The interior walls of an auditorium $250 \times 60 \times 30$ ft^3 have an average absorption coefficient 0·2. The carpeted floor has an absorption coefficient 0·3 and the ceiling has a coefficient 0·4. What is the reverberation time?

(1+5+1)+3=10

★★★

TDP (Honours) 1st Semester Exam., 2019

PHYSICS

(Honours)

FIRST PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **eight** questions, taking **two** from each Unit

UNIT—I

1. *(a)* If $\vec{A}$ and $\vec{B}$ are two irrotational vectors, then show that $(\vec{A} \times \vec{B})$ is also an irrotational vector.

   (b) Prove that if a rigid body is in motion, then the curl of its linear velocity at any point gives twice its angular velocity.

   (c) State Gauss' divergence theorem.

   (d) If $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors, then identify if $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ is a unit vector or not.

2+5+2+1=10

2. *(a)* Show that every square matrix is uniquely expressible as the sum of a Hermitian and skew-Hermitian matrices.

(b) What is the importance of using Fourier series?

(c) If $r(t)$ is a vector of fixed magnitude, then set a relation between $r(\vec{t})$ and $\frac{dr(t)}{dt}$.

(d) Determine the eigenvalue and eigenvector of the following matrix :

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2+1+2+5=10

3. *(a)* Prove that

$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

(b) Use Cayley-Hamilton theorem to find inverse of the matrix

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$$

(c) Write the expression for $\vec{\nabla}$ in curvilinear coordinate system. 4+5+1=10

UNIT—II

4. *(a)* State and prove perpendicular axes theorem of moment of inertia.

   (b) Calculate the MI of a solid sphere about any of its diameter.

   (c) A hollow sphere, a solid sphere, a disc and a ring, all having the same radius and mass, are allowed to roll down an inclined plane. Which one will take *(i)* the shortest and *(ii)* the greatest time to cover a given length? Explain with reason. 2+4+(2+2)=10

5. *(a)* Show that the total energy of a particle of mass m acted upon by a central force is given by

$$E = \frac{h^2}{2m}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] + v(r)$$

   where $v(r)$ is the potential energy, h is the angular momentum and (r, θ) the polar coordinates of the particle and $u = \frac{1}{r}$.

   (b) State and prove Kepler's 3rd law of planetary motion.

   (c) Write the characteristics of motion under central force. 5+3+2=10

6. *(a)* Show that the time derivatives of a vector $\vec{A}$ in a fixed and in a rotating coordinate system are related as

$$\left(\frac{d\vec{A}}{dt}\right)_{\text{fixed}} = \left(\frac{d\vec{A}}{dt}\right)_{\text{rotating}} + \vec{\omega} \times \vec{A}$$

where the symbols have their usual meanings.

(b) What is Coriolis force? Under what condition its value becomes zero and maximum?

(c) Show that if no external torque is applied to a body, then the angular momentum of the body rotating around any axis always remains conserved.

4+(1+1+1)+3=10

UNIT—III

7. *(a)* State Gauss' theorem in gravitation.

(b) What is meant by Gaussian surface? Mention its importance.

(c) Deduce Laplace and Poisson equation for gravitational potential using divergence theorem.

(d) Show the graphical variation of gravitational intensity due to a homogeneous solid sphere with distance from centre of sphere. 1+2+5+2=10

8. *(a)* Explain what is meant by geometrical moment of inertia and internal bending moment of a beam.

(b) Find out an expression for torsional rigidity of a cylinder.

(c) Derive an expression for the terminal velocity of a spherical body falling through a viscous fluid. 3+4+3=10

9. *(a)* Explain how surface tension of liquid can be measured by sessile drop method.

(b) What are the basic assumptions needed for establishing Poiseuille's equation?

(c) Establish Torricelli's theorem. 5+2+3=10

UNIT—IV

10. *(a)* Two mutually perpendicular oscillations are represented by $x(t) = a\sin\omega t$ and $y(t) = b\sin(\omega t + \phi)$. Find the resultant equation. Sketch the Lissajous figure resulting from these oscillations with $\phi = -\pi$ and $\phi = \frac{\pi}{2}$.

(b) Show that if the momentum of a particle executing SHM is plotted against its displacement, an elliptic curve is obtained.

(c) A plane longitudinal wave is propagating along the X direction in an elastic medium. Show that the velocity of the wave is

$$c = \sqrt{\frac{k}{\rho_0}}$$

where k and ρ_0 have their usual meanings. (2+2)+3+3=10

11. (a) A mechanical oscillator of mass m and stiffness constant k is subjected to a damping force proportional to its velocity and an external periodic force $F = F_0 \cos\omega t$. In the steady state, the displacement of the oscillator is given by $x = A\cos(\omega t - \phi)$. Show that, in the steady state, the time-averaged power supplied to the oscillator by the external force equals the time-averaged power dissipated through damping.

(b) State Young-Helmholtz law.

(c) Why does the higher order harmonic decay more sharply for a plucked string than the corresponding harmonic of a strucked string?

(d) In case of forced vibration, what is the phase difference between the applied force and the velocity of the driven system at the natural frequency?

(e) In case of velocity resonance, what will be the phase difference between the applied force and the displacement of driven system? 4+2+2+1+1=10

12. (a) Derive the differential equation of a stretched vibrating string fixed at both ends.

(b) A string of length l, has mass per unit length m. It is under tension T. The string is stuck at $\frac{l}{5}$ and released. What harmonics are excited? Calculate the frequency of the fundamental tone, given $l = 50$ cm, $m = 0 \cdot 0092$ gm/cm, $T = 6$ kg-wt, $g = 9 \cdot 8$ m/sec^2.

(c) Find the ratio of amplitude of 2nd harmonic to the 4th harmonic generated in a strucked string.

(d) Write the Sabine's law regarding acoustics of building. 4+(1+2)+2+1=10

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TDP (Honours) 1st Semester Exam., 2021
(Held in 2022)

PHYSICS

(Honours)

FIRST PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **eight** questions, taking **two** from each Unit

UNIT—I

1. (a) Find the work done in moving an object along a vector $\vec{r} = 3\hat{i} + 2\hat{j} - 5\hat{k}$, if the applied force is $\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$.

(b) Show that the force field defined by

$$\vec{F} = (y^2z^3 - 6xz^2)\hat{i} + 2xyz^3\hat{j} + (3xy^2z^2 - 6x^2z)\hat{k}$$

is a conservative field and hence find the corresponding scalar function.

(c) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, evaluate $\nabla^2 (\ln r)$.

2+(2+3)+3=10

2. (a) Verify the divergence theorem for $\vec{F} = 2x^2y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$, taken over the regions in the first octant bounded by $y^2 + z^2 = 3^2$ and $x = 2$.

(b) What is a Hermitian matrix?

(c) Find the eigenvalues and eigenvectors of the following square matrix :

$$A = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

5+1+4=10

3. (a) Find the unit vector in a cylindrical coordinate system.

(b) Show that $\Gamma(n+1) = n\Gamma(n)$.

(c) Find a series of sines and cosines of multiples of x which will represent $f(x) = x^2$ in the interval $-\pi < x < \pi$.

3+2+5=10

UNIT—II

4. (a) Show that the moment of inertia of a hollow cylinder of inner radius a and outer radius b, mass M and length L about an axis through its centre and perpendicular to its length is $I = M\left[\frac{(a^2+b^2)}{4} + \frac{L^2}{12}\right]$.

(b) Consider a solid sphere of radius R and mass M rolling down a smooth inclined plane making an angle θ with the horizontal. Find an expression for the acceleration of the sphere. What will be the change in acceleration if the sphere is made hollow instead of solid?

(c) State perpendicular axis theorem.

4+(4+1)+1=10

5. (a) Find the tangential and normal components of velocity and acceleration of a particle moving along a curved path in a plane.

(b) Consider a particle of mass m projected with a velocity $\vec{v}$ and at a latitude λ. Calculate the horizontal and vertical components of Coriolis force. 5+5=10

6. (a) Determine the differential equation of motion of a particle moving under central force in plane polar coordinate system.

(b) Discuss with level diagram the nature of orbital motion of a particle moving under a central force when the total energy is *(i)* positive, *(ii)* negative and *(iii)* zero.

(c) State Kepler's third law of planetary motion.

5+3+2=10

UNIT—III

7. (a) Using Gauss theorem, find the potential at any point due to solid sphere. Also, show graphically the variation of potential with the distance from the center of the sphere.

(b) Find an expression for a torsional couple per unit angular twist of a solid cylinder.

(c) Consider two cylinders of the same material having one hollow and one solid. The cylinders have the same length and mass. Compare their torsional rigidity. Take the radius of the solid cylinder is r and for the hollow cylinder inner one is $r/2$ and outer one is $2r$. (4+1)+3+2=10

8. (a) Find the depression of a cantilever beam of uniform cross-section and weight w, when loaded at the free end by a weight w_0. What is the strain energy in this case?

(b) Two soap bubbles of radii 2 cm and 6 cm are joined together so as to form a common surface. Find the radius of that surface.

(c) What is streamline flow? Explain 'two streamline flows cannot intersect'.

4+3+(1+2)=10

9. (a) Show that the excess pressure acting on the curved surface of a curved membrane is given by

$$p = 2T\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

where r_1 and r_2 are the radius of the curvature of the inner and outer surface.

(b) Define Stokes law and also mention the condition of validity of this law.

(c) An oil drop of density $0{\cdot}9013\ \text{g cm}^{-3}$ and radius 3×10^{-4} cm is following through air. What is terminal velocity? Density of air is $0{\cdot}0013\ \text{g cm}^{-3}$, the coefficient velocity of air is 180×10^{-6} CGS unit.

4+(1+2)+3=10

UNIT—IV

10. (a) Show that in case of a particle executing SHM, the average values of KE and PE are equal and each is equal to the half of the total energy.

(b) Show that two SHMs of the same frequency but differ in amplitude and phase when superimposed in the same direction also be simple harmonic. Discuss the outcome of the result.

(Turn Over)

(c) The period of oscillation of a simple pendulum is 2 sec and amplitude is 5°. After 20 complete oscillations, its amplitude is reduced to 4°. Calculate the damping constant of the oscillator.

3+4+3=10

11. (a) Obtain an expression for amplitude and velocity resonance. Draw the graph showing the variation of amplitude and velocity resonance at different frequencies.

(b) Show that the velocity of acoustic waves travelling along a solid rod is $\sqrt{\frac{Y}{\rho}}$, where Y is the Young's modulus and ρ is the density. State the assumptions that are considered to derive the velocity.

(2+2+1)+(4+1)=10

12. (a) A string of length l is fixed at two ends and is plucked at a distance a from one of the fixed ends. Find the expression for displacement and velocity at any point x on the string at any time t.

b) Derive a relationship between group velocity and phase velocity.

(c) State some basic requirements of a good auditorium. 5+3+2=10

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