

MEAN

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The mean (average or arithmetic average) of observed values or data is the sum of the values of the all observed data divided by the total number of observed data. It is a common measure of central tendency and acts as a representative figure for the all observed values or data.

A. MEAN OF UNGROUPED DATA (RAW DATA)

Ungrouped data is raw data or values that have not been categorized into groups under different intervals of same range. The mean of the ungrouped data is calculated using the following formulae:

$$(a) \text{ Mean } (\bar{X}) = (\Sigma X_i)/n$$

Where \bar{X} (X bar) stands for sample mean, Σ (capital sigma) means summation, ΣX_i indicates summation of all individual values of the sample, and n represents number of individuals in the sample.

Example: The resting heart rates (beats per minute or bpm) of 10 men are 78, 60, 82, 77, 67, 72, 74, 80, 67 and 63. Calculate/ Find the mean of the resting heart rates of the men.

Answer:

The mean of the above-mentioned ungrouped data can be calculated using the following formula:

$$\text{Mean } (\bar{X}) = (\Sigma X_i)/n$$

Where \bar{X} (X bar) stands for sample mean, ΣX_i indicates sum of the all individual values of the sample, and n represents number of individuals in the sample

$$\text{Here, } \Sigma X_i = (78+60+82+72+67+72+74+80+67+68)$$

$$=720$$

$$n = 10$$

$$\text{So, mean } (\bar{X}) = 720/10$$

$$= 72$$

Therefore, the mean of the above-mentioned resting rates is 72 bpm.

$$(b) \text{ Mean } (\bar{X}) = (f_1X_1 + f_2X_2 + \dots + f_nX_n) / (f_1 + f_2 + \dots + f_n)$$

$$= \Sigma f_i X_i / \Sigma f_i$$

Where \bar{X} (X bar) is the sample mean; X_1, X_2, \dots, X_n are observed values with respective frequencies f_1, f_2, \dots, f_n ; $\Sigma f_i X_i$ is the sum of the values of the all observations ($f_1X_1 + f_2X_2 + \dots + f_nX_n$); and Σf_i is the sum of the numbers of the all observations ($f_1 + f_2 + \dots + f_n$).

Example: The resting heart rates of 10 men are presented in the table below:

Resting heart rates(bpm) (X_i)	63	67	69	72	74	80	82
Number of men (f_i)	1	2	1	2	2	1	1

Calculate (Find or Compute) the mean of the resting heart rate of the men.

Answer:

The mean of the above-mentioned ungrouped frequency data can be calculated using the following formula:

$$\text{Mean } (\bar{X}) = (f_1X_1 + f_2X_2 + \dots + f_nX_n) / (f_1 + f_2 + \dots + f_n)$$

$$= \Sigma f_i X_i / \Sigma f_i$$

Where \bar{X} is the sample mean; X_1, X_2, \dots, X_n are observed values with respective frequencies f_1, f_2, \dots, f_n ; $\Sigma f_i X_i$ is the sum of the values of the all observations ($f_1X_1 + f_2X_2 + \dots + f_nX_n$); and Σf_i is the sum of the numbers of the all observations ($f_1 + f_2 + \dots + f_n$)

For the above-mentioned data, $\Sigma f_i X_i$ and Σf_i can be calculated in the manner as shown in the following table:

Resting heart rates of men (beats per minute or bpm)(X_i)	Number of men (f_i)	$f_i X_i$
63	1	63
67	2	134
69	1	69
72	2	144
74	2	148
80	1	80
82	1	82
Total	$\Sigma f_i = 10$	$\Sigma f_i X_i = 720$

Here, $\Sigma f_i X_i = 720$, $\Sigma f_i = 10$

So, mean (\bar{X}) = $720/10$

$$= 72$$

Therefore, the mean of the above-mentioned resting rates is 72 bpm.

B. MEAN OF GROUPED DATA

Grouped data are data constituted by arranging individual observed values or data of a variable into groups under different intervals of same range, so that frequency distributions of these groups serve as a convenient way of summarizing the data.

Example: Body weights (kg) of 40 men of a certain village are presented in the table given below:

Class intervals of body weight (kg)	Frequency (i.e., Number of men)
45 – 50	3
50 – 55	5
55 – 60	7
60 – 65	11
65 – 70	5
70 – 75	7
75 – 80	2

Find the mean of the body weights of the village men.

Answer:

1. Direct method

The mean of the above-mentioned grouped frequency data can be calculated using the following formula:

$$\text{Mean } (\bar{X}) = \frac{\sum f_i X_i}{\sum f_i}$$

Where \bar{X} is the sample mean; $\sum f_i X_i$ is the sum of the products of the class mid-points (X_i) and respective frequencies (f_i) of the all class intervals; $\sum f_i$ is the sum of the frequencies (f_i) of the all class intervals.

For the above-mentioned grouped frequency data, $\sum f_i X_i$ and $\sum f_i$ can be calculated in the manner as shown in the following table:

Class intervals of body weight (kg)	Number of men (f_i)	Class mid-point or Class mark (X_i)	$f_i X_i$
45 – 50	3	$(45+50)/2=47.5$	$47.5 \times 3=142.5$
50 – 55	5	52.5	262.5
55 – 60	7	57.5	402.5
60 – 65	11	62.5	687.5
65 – 70	5	67.5	337.5
70 – 75	7	72.5	507.5
75 – 80	2	77.5	155
Total	$\sum f_i = 40$		$\sum f_i X_i = 2495$

Here, $\sum f_i X_i = 2495$, $\sum f_i = 40$

So, mean (\bar{X}) = $2495/40$

$$= 62.375$$

Therefore, the mean of the above-mentioned body weights is 62.375 kg.

This method is referred to as the **direct method**.

2. Assumed mean method

The mean of the above-mentioned grouped frequency data can be calculated using the following formula:

$$\text{Mean } (\bar{X}) = a + (\Sigma f_i d_i / \Sigma f_i)$$

Where \bar{X} is the sample mean; 'a' is the assumed mean; $\Sigma f_i d_i$ is the sum of the products of deviations (d_i) of the class mid-points (from 'a') and respective frequencies (f_i) for the all class intervals; Σf_i is the sum of the frequencies (f_i) of the all class intervals.

For the above-mentioned grouped frequency data, 'a', $\Sigma f_i d_i$ and Σf_i can be calculated in the manner as shown in the following table:

Class interval	Number of men (f_i)	Class mid-point (X_i)	d_i (deviation) = $X_i - a$ (assumed mean)	$f_i d_i$
45 – 50	3	47.5	-15	-45
50 – 55	5	52.5	-10	-50
55 – 60	7	57.5	-5	-35
60 – 65	11	62.5	0	0
65 – 70	5	67.5	5	25
70 – 75	7	72.5	10	70
75 – 80	2	77.5	15	30
Total	$\Sigma f_i = 40$			$\Sigma f_i d_i = -5$

Here, the first step is to select the class mid-point which lies in the centre of the column of X_i (i.e., 62.5) as the **assumed mean**, and to indicate it by 'a'.

The second step is to calculate the difference d_i between 'a' and each class mid-point i.e., the **deviation** (d_i) of 'a' from each class mid-point = $X_i - a = X_i - 62.5$

The third step is to calculate the product of d_i and respective f_i , the sum of the all products ($\Sigma f_i d_i$) and the sum of the all frequencies (Σf_i) as shown in the table.

Here, $a = 62.5$, $\Sigma f_i d_i = -5$, $\Sigma f_i = 40$

So, mean (\bar{X}) = $62.5 + (-5/40)$

$$= 62.5 + (-0.125)$$

$$= 62.375$$

Therefore, the mean of the above-mentioned body weights is 62.375 kg.

This method is referred to as the **assumed mean method**.

3. Step-deviation method

The mean of the above-mentioned grouped frequency data can be calculated using the following formula:

$$\text{Mean } (\bar{X}) = a + h(\Sigma f_i u_i / \Sigma f_i)$$

Where \bar{X} is the sample mean; 'a' is the assumed mean; 'h' is the class size; $\Sigma f_i u_i$ is the sum of the products of step-deviations (u_i) and respective frequencies (f_i) of the all class intervals; Σf_i is the sum of the frequencies (f_i) of the all class intervals.

For the above-mentioned grouped frequency data, 'a', $\Sigma f_i u_i$ and Σf_i can be calculated in the manner as shown in the following table:

Class interval	Number of men (f_i)	Class mid-point (X_i)	d_i (deviation) = $X_i - a$ (assumed mean)	u_i (step-deviation) = $(X_i - a)/h$ (class size)	$f_i u_i$
45 – 50	3	47.5	-15	-3	-9
50 – 55	5	52.5	-10	-2	-10
55 – 60	7	57.5	-5	-1	-7
60 – 65	11	62.5	0	0	0
65 – 70	5	67.5	5	1	5
70 – 75	7	72.5	10	2	14
75 – 80	2	77.5	15	3	6
Total	$\Sigma f_i = 40$				$\Sigma f_i u_i = -1$

Here, the first step is to select the class mid-point which lies in the centre of the column of X_i (i.e., 62.5) as the **assumed mean**, and indicate it by 'a'.

The second step is to calculate the difference d_i between 'a' and each class mid-point i.e., the **deviation** (d_i) of 'a' from each class mid-point = $X_i - a = X_i - 62.5$

The third step is to calculate the step-deviation (u_i), the products of u_i and respective f_i , the sum of the all products ($\Sigma f_i u_i$) and the sum of the all frequencies (Σf_i) as shown in the table.

Here, $a = 62.5$, $h = 5$, $\Sigma f_i u_i = -1$, $\Sigma f_i = 40$

So, mean (\bar{X}) = $62.5 + 5 \times (-1/40)$

$$= 62.5 - 0.125$$

$$= 62.375$$

Therefore, the mean of the above-mentioned body weights is 62.375 kg.

This method is referred to as the **step-deviation method**.